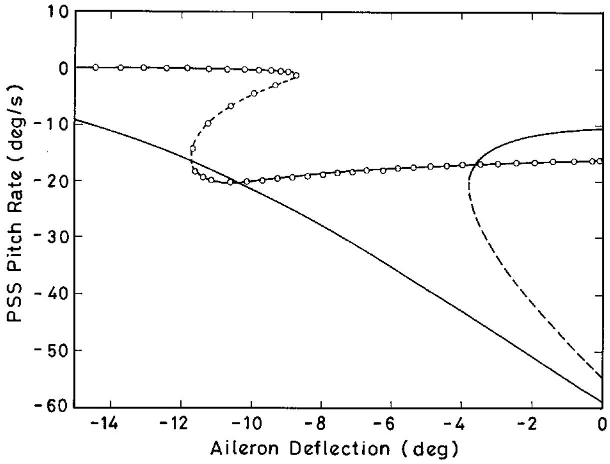

Fig. 3 PSS pitch rate solutions for rolling motion with no constraint (no symbols), and VVR constraint (○). (Both cases: ——, stable solution; - - -, unstable solution.)

pitch rate of -10.5 deg/s, which is the value of q in Fig. 3 corresponding to $p = 0$ in the unconstrained case. Computed PSS solutions for the roll rate and sideslip in this case are also shown in Figs. 1 and 2. The figures show that multiple attractors persist even with the additional pitch rate constraint. The aircraft shows a jump at $\delta a \approx -12$ deg at the saddle-node bifurcation point that occurs for $(\alpha, \beta) \approx (-4, 1)$ deg. It follows that even when the elevator is used to maintain constant pitch rate in a rapid VVR maneuver, the rudder does not suppress sideslip for large δa.

IV. Conclusions

The present study has shown that a VVR strategy does not eliminate multiple attractors and cannot prevent jump in an inertia-coupled rapid roll maneuver. In particular, for large aileron deflections in a VVR, the rudder loses influence over the sideslip and fails to suppress the sideslip.

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Mean Flow Refraction Corrections for the Kirchhoff Method

Anthony R. Pilon*
Pennsylvania State University, University Park, Pennsylvania 16801
and
Anastasios S. Lyrintzis†
Purdue University, West Lafayette, Indiana 47907

Introduction

NOISE generated by supersonic and subsonic jets is important for civil and military aircraft. The success of the High-Speed Civil Transport (HSCT) and other aircraft depends on a substantial reduction of the radiated jet exhaust noise. Thus it is necessary to have accurate jet noise prediction methods so that future aircraft designs can be assessed. One attractive prediction technique is the Kirchhoff method. The Kirchhoff method consists of the calculation of the nonlinear near field, usually numerically. The far-field acoustics are then determined through Kirchhoff's integral formulation, evaluated on a control surface surrounding the nonlinear field.

Kirchhoff's integral equation has recently become popular as a tool for numerical acoustic prediction.[1] Methods based on this integral relation are attractive because they utilize surface integrals, not the volume integrals found in acoustic analogy methods, over a source region to determine far-field acoustics. Additionally, Kirchhoff methods do not suffer the dissipation and dispersion errors found when the mid-field and far-field sounds are directly calculated with an algorithm similar to those used in computational fluid dynamics (CFD) studies.

The Kirchhoff method has been used successfully in the prediction of jet noise by several researchers.[2–4] Shih et al.[5] showed that the Kirchhoff method can predict results nearly identical to those obtained with a direct calculation method, with a substantial savings in CPU time. However, there are some difficulties involved with using the Kirchhoff method, and related methods, for jet aeroacoustic problems. For an accurate prediction the Kirchhoff control surface must completely enclose the aerodynamic source region. This is often difficult or impossible to accomplish with the source regions found in jet acoustic problems. The validity of predictions is also dependent on the control surface being placed in a region where the linear wave equation is valid. Difficulties meeting this criterion frequently arise in jet acoustic studies. Additionally, the existence of steady mean flow gradients outside the Kirchhoff surface will cause refraction of the propagating sound. Failure to account for this refraction will also lead to errors when the observer location is near the jet axis.

This Note outlines the development of corrections to the Kirchhoff method to account for the difficulties caused by mean flow refraction. The corrections are based on geometric acoustic principles, with the steady mean flow approximated as an axisymmetric parallel shear flow. Sample calculations are presented that show the corrections to predict a zone of silence in qualitative agreement with experimental observations. More details can be found in Ref. 6.

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*Research Associate, Department of Aerospace Engineering. Member AIAA.

†Associate Professor, School of Aeronautics and Astronautics. Associate Fellow AIAA.

Kirchhoff Integral Formula

In this section we present the Kirchhoff integral formula for computational aeroacoustics (CAA). Farassat and Myers[7] derived a time-domain Kirchhoff formula valid for moving and deforming surfaces. For simplicity, only the form for stationary surfaces will be shown here. Assume that the linear, homogeneous wave equation is valid for some acoustic variable ϕ, and sound speed a_0, in the entire region outside of a closed and bounded smooth surface S. Then ϕ [with spatial and temporal coordinates (x, t)] for all points in the exterior of S is defined in terms of surface quantities [with coordinates (y, τ)] by

$$4\pi\phi(x, t) = \int_S \frac{1}{r}\left(\frac{1}{a_0}\frac{\partial\phi}{\partial\tau}\cos\theta - \frac{\partial\phi}{\partial n}\right)_\tau \mathrm{d}S + \int_S \frac{(\phi)_\tau}{r^2}\,\mathrm{d}S \tag{1}$$

where $\cos\theta = \hat{r} \times \hat{n}$. The subscript τ in Eq. (1) indicates evaluation of the integrand at the retarded (emission) time $\tau = r/a_0$. With the use of a Fourier transformation, Eq. (1) can be expressed in the frequency domain as

$$4\pi\hat{\phi}(x, \omega) = \int_S e^{i\omega r/a_0}\left[\frac{1}{r}\left(-\frac{i\omega}{a_0}\cos\theta\hat{\phi} - \frac{\partial\hat{\phi}}{\partial n}\right) + \frac{\hat{\phi}\cos\theta}{r^2}\right]\mathrm{d}S \tag{2}$$

where $\hat{\phi}$ is the Fourier transform of ϕ, and ω is the cyclic frequency.

At this point some discussion should be made regarding the choice of acoustic variable ϕ. Most often, the disturbance pressure $p' = p - p_0$ is used for ϕ. The authors feel that the best choice for ϕ lies in the density perturbation ($\phi = a_0^2\rho'$). The authors have shown that when this variable is used the Kirchhoff formula can be reduced to a porous surface form of the Ffowcs Williams-Hawkings formula for noise generated by moving surfaces, plus an additional volume integral of quadrupoles.[8] Thus, nonlinear sound sources at the Kirchhoff surface can be partially accounted for through the use of this variable.

Refraction Effects

The Kirchhoff formulas presented in Eqs. (1) and (2) can efficiently and accurately predict aerodynamically generated noise, as long as the Kirchhoff surface surrounds the entire source region. In jet noise predictions, however, it is usually impossible, with current numerical methods, to determine the entire near-field source region. This is because of time and memory limitations imposed by the computer architecture, as well as dispersion and dissipation constraints. Thus, a significant nonlinear source region, as well as a steady mean flow, will exist outside of the Kirchhoff surface.

The large extent of the source region previously described can be seen in numerical data made available to the authors. The axisymmetric large-scale simulation CFD code of Mankbadi et al.[9] was used to simulate an excited, Mach 2.1, unheated ($T_0 = 294$ K), round jet of Reynolds number $Re = 7 \times 10^4$. The jet exit variables were perturbed in a single axisymmetric mode at a Strouhal number of $Sr = 0.20$. The amplitude of the perturbation was 2% of the mean. The flow data were converted to the frequency domain at all spatial points using a fast Fourier transform algorithm. The disturbance amplitude is quite large at the end of the computational domain. Thus, some approximation of the sources in the region downstream of the Kirchhoff surface is necessary. The authors proposed a suitable approximation in earlier works.[8] In the current work the emphasis will be on the refraction caused by the steady mean flow, and any sources outside the surface will be ignored here. In the future the source approximations will be included along with the refraction corrections presented here.

Even if the unsteady sound sources outside of the Kirchhoff surface can be ignored, there is still a substantial steady mean flow in the region near the jet axis, downstream of the Kirchhoff surface. At the downstream end of the Kirchhoff surface, the mean axial velocity is still over 98% of the jet exit value. The linear wave equation is not valid for acoustic propagation through the region near the jet axis, downstream of the Kirchhoff surface. Thus, some means of approximating the effects of this steady shear flow are required if an acoustic prediction is desired for observer points lying near the jet axis.

Flow Approximation and Effects

A suitable approximation to the downstream shear flow is necessary to determine the refraction effects. In the past, several researchers have used an axisymmetric parallel shear flow model to determine sound produced by point acoustic sources within circular jets.[10] This approach is adopted here as well. More sophisticated approaches, such as those used in Ref. 11, can be used later. A real jet has nonzero radial velocity, but the refracting effect of this component is minimal and can safely be ignored. The numerical simulations used to determine the near-field source terms on the Kirchhoff surface are axisymmetric in nature, so that the lack of azimuthal variation in the parallel shear flow approximation will not have an effect here. The value of the axial velocity to be used in the shear-flow approximation can be taken directly from the near-field numerical simulation, at the downstream end of the Kirchhoff surface, as an average of the time-dependent axial velocity at each radial grid point.

The refraction problem now consists of a collection of point acoustic sources [the integrands of Eq. (2) or Eq. (1)] acting at radial location R, scaled by differential area $\Delta S = R\Delta R\Delta\varsigma$, (where ς is the azimuthal angle), and the parallel shear flow with U determined at each R. If the acoustic wavelength, $\lambda = 2\pi a_0/\omega$, is assumed to be small compared to the shear-layer thickness δ, then geometric acoustics principles hold.

If the steady velocity at the downstream end of the Kirchhoff surface is denoted as U_s, the sound emission angle with respect to the jet axis ϑ_s, and the propagation angle in the stagnant, ambient air is denoted ϑ_0, then the axial acoustic phase speeds are preserved by the stratified flow

$$a_0/\cos\vartheta_0 = U_s + (a_0/\cos\vartheta_s) \tag{3}$$

Here it is assumed that the speed of sound at the source is equivalent to that in the ambient air. This equation can be rearranged to show that there is a critical angle, ϑ_c, defined by

$$\vartheta_c = \cos^{-1}[1/(1 + M_s)] \tag{4}$$

If the observer angle ϑ_0 is greater than ϑ_c, then no sound emitted at the source on the Kirchhoff surface can reach the observer. This criterion is easily added to the stationary surface Kirchhoff program. (Note that M_s is the Mach number of the mean shear flow, and not the Kirchhoff surface, which is assumed stationary.)

An additional correction is necessary to accurately account for the mean flow refraction. Imposing the local zone of silence condition described earlier can allow a surface source at a relatively large radial location to radiate sound into and through the shear flow. This is because the local zone of silence decreases in size with the radial location of the source, because of the decrease in source Mach number. The simple correction is to set the source strength to zero if the observation point is located closer to the jet axis than the source point on the Kirchhoff surface.

It should be noted that the azimuthal variation between the source and observer points has been ignored in the analysis presented here. The azimuthal variation should have some effect, but it is most likely secondary to those effects described earlier. (Though the near-field CFD calculations are axisymmetric, the Kirchhoff surface is a full three-dimensional cylinder, and so discrepancies between source and observer azimuthal location can exist.) Also, the geometric acoustics approximation is only valid for $\delta/\lambda > 1$. It is assumed here that the downstream end of the cylindrical Kirchhoff surface is located far enough downstream of the jet potential core that the shear-layer thickness is large compared with the acoustic wavelength.

Jet Noise Calculation

The axisymmetric near-field jet CFD calculations discussed earlier were used to determine the integrands in the Kirchhoff integral formula, and also to predict the parallel shear flow downstream of the Kirchhoff surface. The surface was chosen to match lines in the mesh used for the CFD calculations, so that $L_k = 64.67R_j$. (The surface extended axially from $x = 5R_j$ to $x = 69.67R_j$.) The cylindrical Kirchhoff surface had a radius of $R_k = 8.56R_j$. These values were deemed to be the best choices among the available data, based on mesh spacing and the linearity of disturbances near the surface. However, it is evident that the disturbance amplitudes are still quite large at the downstream end of the surface, and so a much longer Kirchhoff surface would be preferable. However, the extensive CFD calculations needed to determine the acoustic quantities on a longer surface would lead to prohibitive CPU times. There were 389 axial, 167 radial, and 90 azimuthal quadrature points on the Kirchhoff surface. The radial mesh was exponentially stretched about $R = R_j$. Midpoint quadrature was used in the determination of the integral solutions in the frequency domain formulation of the Kirchhoff method. The time step used in the CFD calculations corresponds to 1/64 of the period of the excitation frequency. Because of constraints imposed by the number of spatial points per wavelength required for an accurate prediction, only the first four Fourier modes are used in the Kirchhoff predictions.

The effect of the refraction corrections on this jet noise prediction is shown in Fig. 1. The figure shows instantaneous contours of $a_0^2\rho'/p_0$ on a plane passing through the jet axis, calculated using the numerical data described earlier. The contours shown above the jet axis are those obtained when the mean flow refraction effects were ignored. The contours shown below the jet axis were calculated in an identical fashion, except that the effects of mean flow refraction were included in sound generated at the downstream end of the Kirchhoff surface. Both calculations capture the Mach wave radiation in the region $R > 10R_j$ identically. The steady mean shear flow has little effect on the radiation in this region. Downstream of the Kirchhoff surface, the sound waves appear to propagate spherically away from an equivalent source located near $x \approx 30R_j$. In the prediction without refraction correction the sound waves have large amplitude near the end of the Kirchhoff surface and propagate as through a uniform stationary medium. The corrections, however, reduce the amplitude in the region near the jet axis and adjust the phase of each disturbance. The corrected sound waves propagate away from the axis at a modest angle. This creates a zone of silence near the axis, similar in nature to those observed experimentally. Also noteworthy is the prediction of sound inside the Kirchhoff surface. A null acoustic field should be calculated inside the surface. The sound field inside the surface shown here is a result of several factors. Among these factors are numerical roundoff errors and the interpolation routine used by the graphics program. Also, the upstream end of the Kirchhoff surface was left open in the predictions.

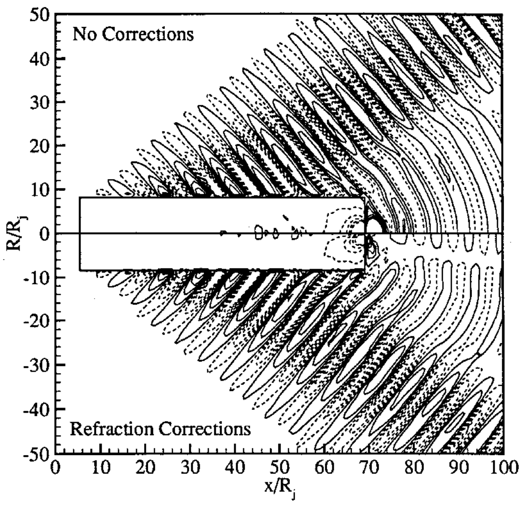

Fig. 1 Instantaneous contours of $a_0^2\rho'/p_0$, $R > 0$: no refraction corrections; $R < 0$: refraction corrections imposed.

In the past researchers utilizing Kirchhoff methods to predict jet noise have ignored sound generated at and outside of the downstream end of the Kirchhoff surface.[2–4] If the observer lies in an area in which a majority of the sound is predicted by the constant radius portion of the Kirchhoff surface, then this omission may not pose a problem. However, the authors have shown that open surface Kirchhoff methods are not acceptable for jet acoustic predictions when the observer is in the region downstream of the Kirchhoff surface. This is consistent with the findings in Ref. 12. The refraction corrections presented here can aid in the accurate prediction of sound in this region.

Conclusions

This paper has presented results of the initial development of refraction corrections for use with the Kirchhoff method. The corrections were developed in an effort to enhance the prediction capabilities of the Kirchhoff method in jet noise studies. In the past the Kirchhoff method has proven to be an efficient and accurate tool for the prediction of jet noise and other aeroacoustic phenomena. It is hoped that, with further development, the simple corrections derived here will allow for accurate prediction of the zone of silence and downstream jet acoustic radiation fields. These capabilities are currently not available with the Kirchhoff method. The corrections are based on geometric acoustic principles and parallel shear flow assumptions. In the future these assumptions will be relaxed to improve the accuracy of the Kirchhoff predictions. The corrections will also be employed in conjunction with Kirchhoff methods for acoustics in two dimensions, Kirchhoff methods for moving surfaces, and approximations to the volume integral found when acoustic sources exist outside of the Kirchhoff surface.

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